Probabilistic Modeling of Signature Using the Method of Hurwitz-Radon Matrices

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Abstract

Artificial intelligence needs methods for object modeling having the set of key points. A novel method of Hurwitz-Radon Matrices (MHR) is used in signature modeling. Proposed method is based on the family of Hurwitz-Radon (HR) matrices which possess columns composed of orthogonal vectors. Two-dimensional curve as signature is modeling via different functions as probability distribution functions: polynomial, sinus, cosine, tangent, cotangent, logarithm, exponent, arc sin, arc cos, arc tan and power function. It is shown how to build the orthogonal matrix OHR operator and how to use it in a process of signature modeling.

Keywords

Signature Modeling; Hurwitz-Radon Matrices; Coefficient of MHR Method; Probabilistic Modeling

Introduction

Signature description by key points is a critical part in many applications of image processing. Thus artificial intelligence have a problem: how to model the curve [1,2] via discrete set of two-dimensional points? Also the matter of signature representation and description is still opened [3,4]. The author wants to approach a problem of signature representation by characteristic points. Proposed method relies on functional modeling of curve points situated between the basic set of the nodes. The functions that are used in calculations represent whole family of elementary functions: polynomials, trigonometric, cyclometric, logarithmic, exponential and power function. These functions are treated as probability distribution functions in the range [0;1]. Nowadays methods apply mainly polynomial functions, for example Bernstein polynomials in Bezier curves, splines and NURBS [5]. Numerical methods for data interpolation are based on polynomial or trigonometric functions, for example Lagrange, Newton, Aitken and Hermite methods. These methods have some weak sides [6] and are not sufficient for object modeling in the situations when the signature cannot be build by polynomials or trigonometric functions. Also trigonometric basis functions in Fourier Series Shape Models are not appropriate for describing all shapes. Model-based vision such as Active Contour Models (called Snakes) or Active Shape Models use the training sets to fit the data and they are applied only for closed curves. In this paper discussed approach is not limited to closed curves and it does not use a training set of some images, but only a set of two-dimensional nodes of the signature. Proposed signature modeling is the functional modeling via any elementary functions and it helps us to fit the curve and to match the signature in object modeling or image analysis.

The author presents novel method of flexible signature modeling. This paper takes up new method of two-dimensional curve modeling via using a family of Hurwitz-Radon matrices. The method of Hurwitz-Radon Matrices (MHR) requires minimal assumptions about object. The only information about shape or curve is the set of at least five nodes. Proposed method of Hurwitz-Radon Matrices (MHR) is applied in curve modeling via different coefficients: polynomial, sinusoidal, cosinusoidal, tangent, cotangent, logarithmic, exponential, arc sin, arc cos, arc tan or power. Function for coefficient calculations is chosen individually at each signature modeling and it represents probability distribution function. MHR method uses two-dimensional vectors (x,y) for curve modeling. Shape of the object is represented by succeeding points $(x_i,y_i) \in \mathbf{R}^2$ as follows in MHR method:

- MHR version with no matrices ($N = 1$) needs 2 nodes or more;
- At least five nodes (x_1,y_1), (x_2,y_2), (x_3,y_3), (x_4,y_4) and (x_5,y_5) if MHR method is implemented with matrices of dimension $N = 2$;
- For better modeling nodes ought to be settled at key points of the curve, for example local minimum or maximum and at least one point between two successive local extrema.

Condition 2 is connected with important features of MHR method: MHR version with matrices of dimension $N = 2$ (MHR2) requires at least five nodes, MHR version with matrices of dimension $N = 4$ (MHR4) needs at least nine nodes and MHR version with matrices of dimension $N = 8$ (MHR8) requires at least 17 nodes. Condition 3 means for example the highest point of the object in a particular orientation, convexity changing or curvature extrema. So this paper wants to answer the question: how to model the signature for discrete set of points?

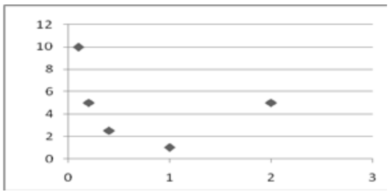

FIG. 1 NODES OF THE SIGNATURE BEFORE MODELING

Coefficients for signature modeling are computed via individual features of the object using probability distribution functions: polynomials, power functions, sinus, cosine, tangent, cotangent, logarithm, exponent or arc sin, arc cos, arc tan.

Probabilistic Modeling

The method of Hurwitz – Radon Matrices (MHR), described in this paper, is computing points between two successive nodes of the curve. Signature data are interpolated and parameterized for real number $\alpha \in [0;1]$ in the range of two successive nodes. MHR calculations are introduced with square matrices of dimension $N = 1, 2, 4$ or 8. Matrices A_i, $i = 1,2 \ldots m$ satisfying

$$A_j A_k + A_k A_j = 0, \qquad A_j^2 = -I \qquad \text{for } j \neq k;\ j, k = 1,2\ldots m$$

are called *a family of Hurwitz - Radon matrices*, discussed by Adolf Hurwitz and Johann Radon separately in 1923. A family of Hurwitz - Radon (HR) matrices [7] are skew-symmetric ($A_i^T = -A_i$), $A_i^{-1} = -A_i$ and only for dimension $N = 1, 2, 4$ or 8 the family of HR matrices consists of $N - 1$ matrices. For $N = 1$ there is no matrices but only calculations with real numbers. For $N = 2$:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

For $N = 4$ there are three HR matrices with integer entries:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}.$$

For $N = 8$ we have seven HR matrices with elements 0, ±1. So far HR matrices have found applications in Space-Time Block Coding (STBC) [8] and orthogonal design [9], in signal processing [10] and Hamiltonian Neural Nets [11].

How coordinates of curve points are applied in Active Object Modeling? If boundary points have the set of following nodes $\{(x_i,y_i),\ i = 1, 2, \ldots, n\}$ then HR matrices combined with the identity matrix I_N are used to build the orthogonal Hurwitz - Radon Operator (OHR). For point $p_1=(x_1,y_1)$ OHR of dimension $N = 1$ is represented by matrix (real number) M_1:

$$M_1(p_1) = \frac{1}{x_1^2}[x_1 \cdot y_1] = \frac{y_1}{x_1}. \tag{1}$$

For points $p_1=(x_1,y_1)$ and $p_2=(x_2,y_2)$ OHR of dimension $N=2$ is build via matrix M_2:

$$M_2(p_1, p_2) = \frac{1}{x_1^2 + x_2^2}\begin{bmatrix} x_1y_1 + x_2y_2 & x_2y_1 - x_1y_2 \\ x_1y_2 - x_2y_1 & x_1y_1 + x_2y_2 \end{bmatrix}. \tag{2}$$

For points $p_1=(x_1,y_1)$, $p_2=(x_2,y_2)$, $p_3=(x_3,y_3)$ and $p_4=(x_4,y_4)$ OHR M_4 of dimension $N = 4$ is introduced:

$$M_4(p_1, p_2, p_3, p_4) = \frac{1}{x_1^2 + x_2^2 + x_3^2 + x_4^2}\begin{bmatrix} u_0 & u_1 & u_2 & u_3 \\ -u_1 & u_0 & -u_3 & u_2 \\ -u_2 & u_3 & u_0 & -u_1 \\ -u_3 & -u_2 & u_1 & u_0 \end{bmatrix} \tag{3}$$

where

$$u_0 = x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4, \quad u_1 = -x_1y_2 + x_2y_1 + x_3y_4 - x_4y_3,$$
$$u_2 = -x_1y_3 - x_2y_4 + x_3y_1 + x_4y_2, \quad u_3 = -x_1y_4 + x_2y_3 - x_3y_2 + x_4y_1.$$

For nodes $p_1=(x_1,y_1)$, $p_2=(x_2,y_2)$,… and $p_8=(x_8,y_8)$ OHR M_8 of dimension $N = 8$ is constructed [12] similarly as (1) and (2):

$$M_8(p_1, p_2 \ldots p_8) = \frac{1}{\sum_{i=1}^{8} x_i^2}\begin{bmatrix} u_0 & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 \\ -u_1 & u_0 & u_3 & -u_2 & u_5 & -u_4 & -u_7 & u_6 \\ -u_2 & -u_3 & u_0 & u_1 & u_6 & u_7 & -u_4 & -u_5 \\ -u_3 & u_2 & -u_1 & u_0 & u_7 & -u_6 & u_5 & -u_4 \\ -u_4 & -u_5 & -u_6 & -u_7 & u_0 & u_1 & u_2 & u_3 \\ -u_5 & u_4 & -u_7 & u_6 & -u_1 & u_0 & -u_3 & u_2 \\ -u_6 & u_7 & u_4 & -u_5 & -u_2 & u_3 & u_0 & -u_1 \\ -u_7 & -u_6 & u_5 & u_4 & -u_3 & -u_2 & u_1 & u_0 \end{bmatrix} \tag{4}$$

where

$$\underline{u} = \begin{bmatrix} y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_7 & y_8 \\ -y_2 & y_1 & -y_4 & y_3 & -y_6 & y_5 & y_8 & -y_7 \\ -y_3 & y_4 & y_1 & -y_2 & -y_7 & -y_8 & y_5 & y_6 \\ -y_4 & -y_3 & y_2 & y_1 & -y_8 & y_7 & -y_6 & y_5 \\ -y_5 & y_6 & y_7 & y_8 & y_1 & -y_2 & -y_3 & -y_4 \\ -y_6 & -y_5 & y_8 & -y_7 & y_2 & y_1 & y_4 & -y_3 \\ -y_7 & -y_8 & -y_5 & y_6 & y_3 & -y_4 & y_1 & y_2 \\ -y_8 & y_7 & -y_6 & -y_5 & y_4 & y_3 & -y_2 & y_1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} \tag{5}$$

and $\underline{u} = (u_0, u_1, \ldots, u_7)^T$ (4). OHR operators M_N (0)-(3) satisfy the condition of interpolation

$$M_N \cdot x = y \tag{6}$$

for $\mathbf{x} = (x_1,x_2\ldots,x_N)^T \in \mathbf{R}^N$, $\mathbf{x} \neq \mathbf{0}$, $\mathbf{y} = (y_1,y_2\ldots,y_N)^T \in \mathbf{R}^N$ and $N = 1, 2, 4$ or 8.

Distribution Functions in MHR Probabilistic Modeling

Coordinates of points settled between the nodes are computed [13] using described MHR method [14]. Each real number $c \in [a;b]$ is calculated by a convex combination $c = \alpha \cdot a + (1 - \alpha) \cdot b$ for

$$\alpha = \frac{b-c}{b-a} \in [0;1]. \tag{7}$$

The average OHR operator M of dimension $N = 1, 2, 4$ or 8 is build:

$$M = \gamma \cdot A + (1-\gamma) \cdot B \cdot \tag{8}$$

The OHR matrix A is constructed (1)-(3) by every second point $p_1=(x_1=a,y_1)$, $p_3=(x_3,y_3)$,…and $p_{2N-1}=(x_{2N-1}, y_{2N-1})$:

$$A = M_N(p_1, p_3,\ldots, p_{2N-1}).$$

The OHR matrix B is computed (1)-(3) by data $p_2=(x_2=b,y_2)$, $p_4=(x_4,y_4)$,… and $p_{2N}=(x_{2N},y_{2N})$:

$$B = M_N(p_2, p_4,\ldots, p_{2N}).$$

Vector of first coordinates C is defined for

$$c_i = \alpha \cdot x_{2i-1} + (1-\alpha) \cdot x_{2i} \quad , \quad i = 1, 2,\ldots, N \tag{9}$$

and $C = [c_1, c_2,\ldots, c_N]^T$. The formula to calculate second coordinates $y(c_i)$ is similar to the interpolation formula (5):

$$Y(C) = M \cdot C \tag{10}$$

where $Y(C) = [y(c_1), y(c_2),\ldots, y(c_N)]^T$. So modeled value of $y(c_i)$ from (9) depends on two, four, eight or sixteen ($2N$) successive nodes. For example $N = 1$ results in computations without matrices:

$$A = M_1(p_1) = \frac{y_1}{x_1},\ B = M_1(p_2) = \frac{y_2}{x_2},\ C = c_1 = \alpha \cdot x_1 + (1-\alpha) \cdot x_2\ ,$$

$$Y(C) = y(c_1) = (\gamma \frac{y_1}{x_1} + (1-\gamma)\frac{y_2}{x_2}) \cdot c_1,$$

$$y(c_1) = \alpha \cdot \gamma \cdot y_1 + (1-\alpha)(1-\gamma) y_2 + \gamma(1-\alpha)\frac{y_1}{x_1} x_2 + \alpha(1-\gamma)\frac{y_2}{x_2} x_1 \tag{11}$$

Key question is dealing with coefficient γ in (7). Basic MHR version means $\gamma = \alpha$ and then (10):

$$y(c_1) = \alpha^2 \cdot y_1 + (1-\alpha)^2 y_2 + \alpha(1-\alpha)(\frac{y_1}{x_1} x_2 + \frac{y_2}{x_2} x_1). \tag{12}$$

Formula (11) differs from linear interpolation:

$$y(c) = \alpha \cdot y_1 + (1-\alpha) y_2.$$

MHR is not a linear interpolation.

Each individual signature requires specific distribution of parameter γ (7) and γ depends on parameter $\alpha \in [0;1]$:

$$\gamma = F(\alpha),\ F(0) = 0,\ F(1) = 1$$

and F is strictly monotonic.

Coefficient γ is calculated using different functions (polynomials, power functions, sinus, cosine, tangent, cotangent, logarithm, exponent, arc sin, arc cos, arc tan) and choice of function is connected with initial requirements and signature specifications during fitting and matching of the object. Coefficients γ and α (6) are strongly related:

- $\gamma = 0 \leftrightarrow \alpha = 0$;
- $\gamma = 1 \leftrightarrow \alpha = 1$;
- $\gamma \in [0;1]$.

Different values of coefficient γ are connected with applied functions $F(\alpha)$. These functions (12)-(38) represent the probability distribution functions for random variable $\alpha \in [0;1]$ and real number $s > 0$:

power function

$$\gamma = \alpha^s \quad \text{with} \quad s > 0. \tag{13}$$

For $s = 1$: basic version of MHR method when $\gamma = \alpha$.

sinus

$$\gamma = \sin(\alpha^s \cdot \pi/2)\ ,\ s > 0 \tag{14}$$

or

$$\gamma = \sin^s(\alpha \cdot \pi/2)\ ,\ s > 0. \tag{15}$$

For $s = 1$: $\gamma = \sin(\alpha \cdot \pi/2)$. (16)

cosine

$$\gamma = 1-\cos(\alpha^s \cdot \pi/2)\ ,\ s > 0 \tag{17}$$

or

$$\gamma = 1-\cos^s(\alpha \cdot \pi/2)\ ,\ s > 0. \tag{18}$$

For $s = 1$: $\gamma = 1-\cos(\alpha \cdot \pi/2)$. (19)

tangent

$$\gamma = \tan(\alpha^s \cdot \pi/4)\ ,\ s > 0 \tag{20}$$

or

$$\gamma = \tan^s(\alpha \cdot \pi/4)\ ,\ s > 0. \tag{21}$$

For $s = 1$: $\gamma = \tan(\alpha \cdot \pi/4)$. (22)

logarithm

$$\gamma = \log_2(\alpha^s + 1)\ ,\ s > 0 \tag{23}$$

or

$$\gamma = \log_2^s(\alpha + 1)\ ,\ s > 0. \tag{24}$$

For $s = 1$: $\gamma = \log_2(\alpha + 1)$. (25)

exponent

$$\gamma = (\frac{a^\alpha - 1}{a-1})^s\ ,\ s > 0 \text{ and } a > 0 \text{ and } a \neq 1. \tag{26}$$

For $s = 1$ and $a = 2$: $\gamma = 2^{\alpha} - 1$. (27)

arcsine

$$\gamma = 2/\pi \cdot arcsin(\alpha^s)\,,\ s > 0 \quad (28)$$

or

$$\gamma = (2/\pi \cdot arcsin\ \alpha)^s\,,\ s > 0. \quad (29)$$

For $s = 1$: $\gamma = 2/\pi \cdot arcsin(\alpha)$. (30)

arccosine

$$\gamma = 1\text{-}2/\pi \cdot arccos(\alpha^s)\,,\ s > 0 \quad (31)$$

or

$$\gamma = 1\text{-}(2/\pi \cdot arccos\ \alpha)^s\,,\ s > 0. \quad (32)$$

For $s = 1$: $\gamma = 1\text{-}2/\pi \cdot arccos(\alpha)$. (33)

arctangent

$$\gamma = 4/\pi \cdot arctan(\alpha^s)\,,\ s > 0 \quad (34)$$

or

$$\gamma = (4/\pi \cdot arctan\ \alpha)^s\,,\ s > 0. \quad (35)$$

For $s = 1$: $\gamma = 4/\pi \cdot arctan(\alpha)$. (36)

cotangent

$$\gamma = ctg(\pi/2 - \alpha^s \cdot \pi/4)\,,\ s > 0 \quad (37)$$

or

$$\gamma = ctg^s\,(\pi/2 - \alpha \cdot \pi/4),\ s > 0. \quad (38)$$

For $s = 1$: $\gamma = ctg(\pi/2 - \alpha \cdot \pi/4)$. (39)

Functions used in γ calculations (12)-(38) are strictly monotonic for random variable $\alpha \in [0;1]$ as $\gamma = F(\alpha)$ - probability distribution function. Choice of function and value s depends on object specifications and individual requirements. Fixing of unknown coordinates for curve points using (6)-(9) is called by author the method of Hurwitz - Radon Matrices (MHR) [15]. So here are five steps of MHR modeling:

- Choice of nodes at key points.
- Fixing the dimension of matrices N = 1, 2, 4 or 8: N = 1 is the most universal for calculations (it needs only two successive nodes to compute unknown point between them) and it has the lowest computational costs (10); MHR with N = 2 uses four successive nodes to compute unknown coordinate; MHR version for N = 4 applies eight successive nodes to get unknown point and MHR with N = 8 needs sixteen successive nodes to calculate unknown coordinate (it has the biggest computational costs).
- Choice of distribution $\gamma = F(\alpha)$: basic distribution for $\gamma = \alpha$.
- Determining values of α: α = 0.1, 0.2…0.9 or 0.01, 0.02…0.99 or others.
- The computations (9).

These five steps can be treated as the algorithm of MHR method of modeling and interpolation (6)-(9).

Considering nowadays used probability distribution functions for random variable $\alpha \in [0;1]$ - one distribution is dealing with the range [0;1]: beta distribution. Probability density function f for random variable $\alpha \in [0;1]$ is:

$$f(\alpha) = c \cdot \alpha^s \cdot (1-\alpha)^r\ ,\ s \geq 0,\ r \geq 0 \quad (40)$$

When $r = 0$ probability density function (39) represents $f(\alpha) = c \cdot \alpha^s$ and then probability distribution function F is like (12), for example $f(\alpha) = 3\alpha^2$ and $\gamma = \alpha^3$. If s and r are positive integer numbers then γ is the polynomial, for example $f(\alpha) = 6\alpha(1-\alpha)$ and $\gamma = 3\alpha^2 - 2\alpha^3$. So beta distribution gives us coefficient γ in (7) as polynomial because of interdependence between probability density f and distribution F functions:

$$f(\alpha) = F'(\alpha)\ ,\ F(\alpha) = \int_0^{\alpha} f(t)dt\,. \quad (41)$$

For example (40): $f(\alpha) = \alpha \cdot e^{\alpha}$ and $\gamma = F(\alpha) = (\alpha - 1)e^{\alpha} + 1$.

What is very important: two signatures may have the same set of nodes but different N or γ results in different signatures (Fig.6-13). Here are some applications of MHR method with basic version ($\gamma=\alpha$): MHR2 is MHR version with matrices of dimension N=2 and MHR4 means MHR version with matrices of dimension N = 4.

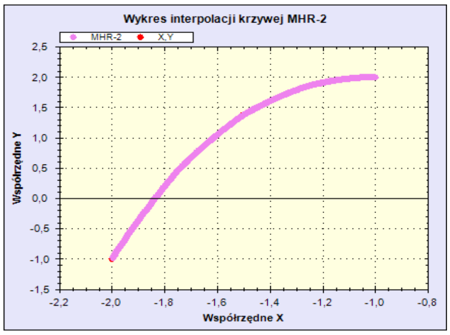

FIG. 2 FUNCTION $F(X) = X^3+X^2-X+1$ WITH 396 INTERPOLATED POINTS USING BASIC MHR2 WITH 5 NODES

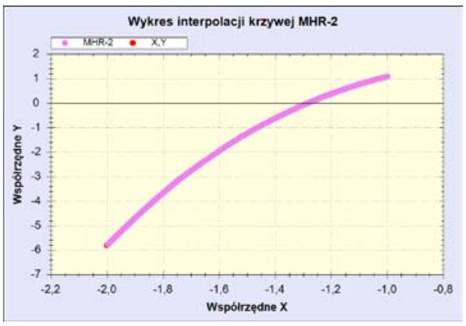

FIG. 3 FUNCTION $F(X) = X^3+LN(7-X)$ WITH 396 INTERPOLATED POINTS USING BASIC MHR2 WITH 5 NODES

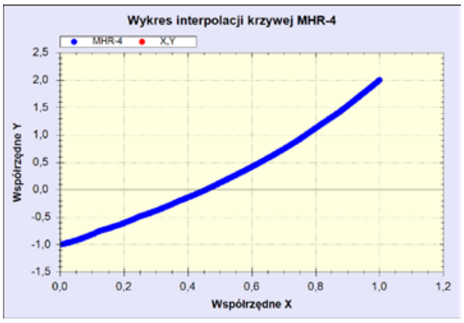

FIG. 4 FUNCTION $F(X) = X^3+2X-1$ WITH 792 INTERPOLATED POINTS USING BASIC MHR4 WITH 9 NODES

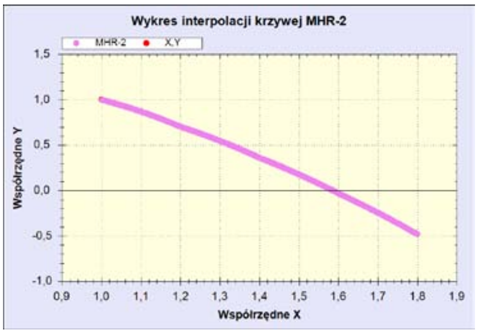

FIG. 5 FUNCTION $F(X) = 3-2^X$ WITH 396 INTERPOLATED POINTS USING BASIC MHR2 WITH 5 NODES

Figures 2-5 show modeling of continues data connected with determined functions. So these functions are modeled. Without knowledge about function, signature modeling has to implement the coefficients γ (12)-(40), but MHR is not limited only to these coefficients. Each strictly monotonic function F between points (0;0) and (1;1) can be used in MHR signature modeling.

Applications of Mhr Probabilistic Modeling

Signature nodes: (0.1;10), (0.2;5), (0.4;2.5), (1;1) and (2;5) from Fig.1 are used in some examples of MHR signature modeling with different γ. Points of the signature are calculated for $N = 1$ and $\gamma = \alpha$ (11) in example 1 and with matrices of dimension N=2 in examples 2 - 8 for $\alpha = 0.1, 0.2,\ldots,0.9$.

Signature modeling for $N = 1$ and $\gamma = \alpha$.

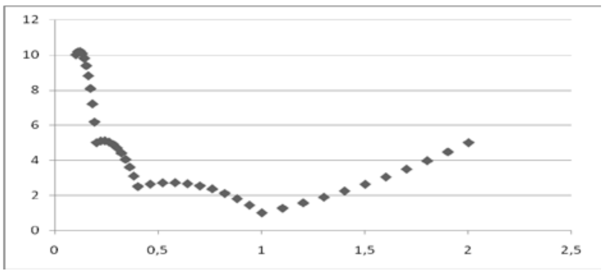

FIG. 6 MODELING WITHOUT MATRICES ($N = 1$) FOR NINE RECONSTRUCTED POINTS BETWEEN NODES

Sinusoidal signature modeling with $\gamma = sin(\alpha \cdot \pi/2)$.

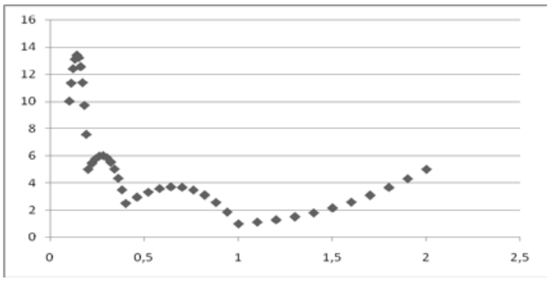

FIG. 7 SINUSOIDAL MODELING WITH NINE RECONSTRUCTED SIGNATURE POINTS BETWEEN NODES

Tangent modeling for $\gamma = tan(\alpha \cdot \pi/4)$.

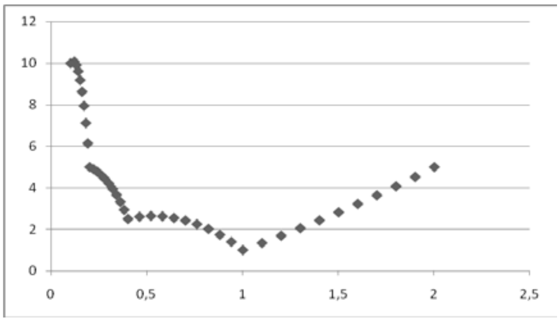

FIG. 8 TANGENT SIGNATURE MODELING WITH NINE INTERPOLATED POINTS BETWEEN NODES

Tangent modeling with $\gamma = tan(\alpha^k \cdot \pi/4)$ and $k = 1.5$.

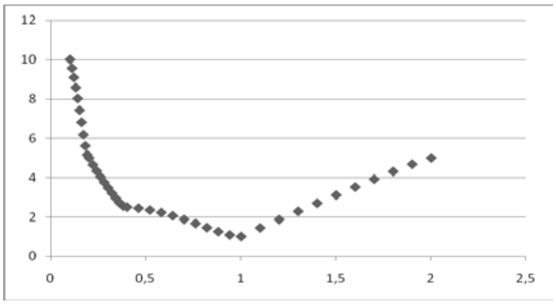

FIG. 9 TANGENT MODELING WITH NINE RECOVERED SIGNATURE POINTS BETWEEN NODES

Tangent signature modeling for $\gamma = tan(\alpha^k \cdot \pi/4)$ and $k = 1.797$.

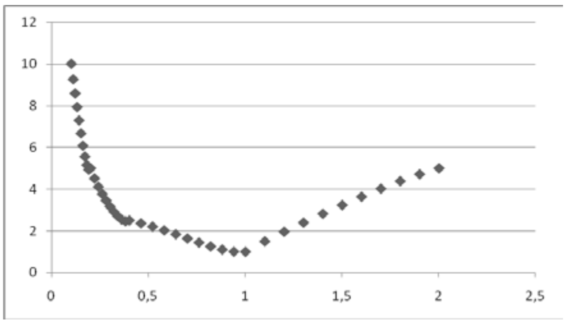

FIG. 10 TANGENT MODELING WITH NINE RECONSTRUCTED POINTS BETWEEN NODES

Sinusoidal modeling with $\gamma = sin(\alpha^k \cdot \pi/2)$ and $k = 2.759$.

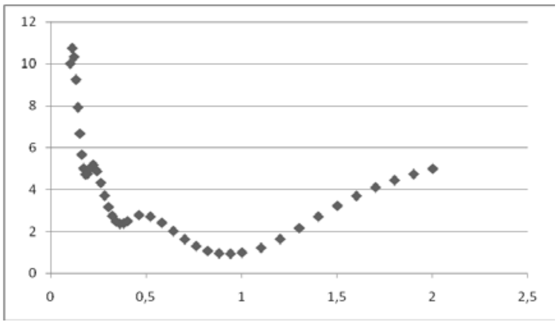

FIG. 11 SINUSOIDAL MODELING WITH NINE INTERPOLATED SIGNATURE POINTS BETWEEN NODES

Power function modeling for $\gamma = \alpha^k$ and $k = 2.1205$.

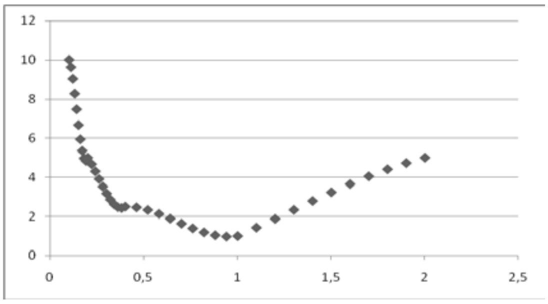

FIG. 12 POWER FUNCTION SIGNATURE MODELING WITH NINE RECOVERED OBJECT POINTS BETWEEN NODES

Logarithmic signature modeling with $\gamma = log_2(\alpha^k + 1)$ and $k = 2.533$.

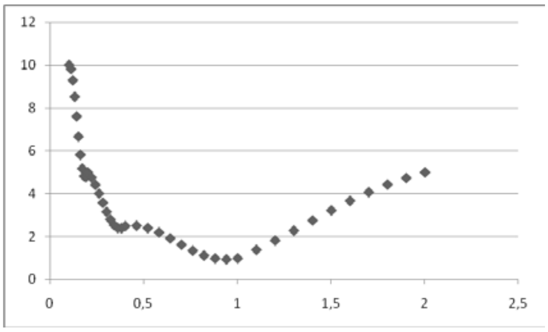

FIG. 13 LOGARITHMIC MODELING WITH NINE RECONSTRUCTED POINTS BETWEEN NODES

These eight examples demonstrate possibilities of signature modeling for key nodes. Reconstructed values and interpolated points, calculated by MHR method, are applied in the process of curve modeling for fitting and matching the signature during its analysis. Every individual signature or handwriting, each letter or number can be modeled by some function for parameter γ. This parameter is treated as probability distribution function for each individual signature

Conclusions

The method of Hurwitz-Radon Matrices (MHR) enables modeling of two-dimensional curves using different coefficients γ: polynomial, sinusoidal, cosinusoidal, tangent, cotangent, logarithmic, exponential, arc sin, arc cos, arc tan or power function [16]. Function for γ calculations is chosen individually at each signature modeling and it is treated as probability distribution function: γ depends on initial requirements and curve specifications. MHR method leads to signature modeling via discrete set of fixed points. So MHR makes possible the combination of two important problems: interpolation and modeling. Main features of MHR method are:

- modeling of L points is connected with the computational cost of rank $O(L)$;
- MHR is well-conditioned method (orthogonal matrices) [17];
- coefficient γ is crucial in the process of signature modeling and it is computed individually for each object (letter or figure).

Future works are going to: features of coefficient γ, implementation of MHR in object recognition [18], shape representation, curve fitting, contour modeling and parameterization [19].

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